Hall effect at a tunable metal-insulator transition

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Using a rotating magnetic field, the Hall effect in three-dimensional amorphous Gd$_x$Si$_{1-x}$ has been measured in the critical regime of the metal-insulator transition for a constant total magnetic field. The Hall coefficient $R_0$ is negative, indicating electronlike conductivity, with a magnitude that increases with decreasing conductivity. $R_0$ diverges at the metal-insulator transition, and displays critical behavior with exponent $-1/2 \left[ R_0 \sim (H - H_c)^{-1} \right]$. This dependence is interpreted as a linear decrease in the density of mobile carriers $n \sim R_0 \sim H - H_c$, indicative of the dominant influence of interaction effects.

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Investigation of the electronic properties at the metal-insulator transition (MIT) is a topic of current interest because the MIT displays characteristics of a $T=0$ K quantum phase transition.\textsuperscript{4} To probe the MIT in the critical regime, it is advantageous to be able to control the transition in a continuous way. Most previous studies have examined a series of samples of varying concentration spanning the MIT, introducing uncertainties due to possible inhomogeneity or irreproducibility in the sample preparation process. An elegant solution to these shortcomings arises from using a single sample, which is controlled by an external parameter such as stress,\textsuperscript{5} magnetic field\textsuperscript{4} or illumination,\textsuperscript{4} thereby substantially improving the quality of transport data and allowing measurements of, e.g., the electronic density of states by electron tunneling,\textsuperscript{5} or the dielectric constant.\textsuperscript{6}

The issue addressed in the present work is the critical behavior of the density of mobile carriers as measured by the Hall effect. While it has already been shown from tunneling experiments that the single-particle density of states vanishes at the MIT,\textsuperscript{7} in a correlated electron material such as exists at the MIT, the mobile carrier density is distinct from the single particle electron density of states. According to Altshuler and co-workers,\textsuperscript{7} a system that is dominated by electron-electron (e-e) interactions should exhibit critical behavior for the Hall coefficient $R_0$. Fukuyama has suggested\textsuperscript{8} that corrections due to localization should leave $R_0$ unchanged, a result consistent with calculations (in the absence of electron interactions) by Shapiro and Abrahams.\textsuperscript{9}

In the two-dimensional (2D) disorder-induced MIT, it has been found theoretically that the Hall coefficient is critical with exponent $-1$.\textsuperscript{10} The material used in this study has a large carrier concentration (it is whether these carriers are mobile or not that we are investigating) at the MIT and therefore should exhibit strong e-e interactions and localization effects. Previous studies of the Hall effect near the MIT for crystalline Ge:Sb and for amorphous Kr-Bi alloys show a divergent Hall coefficient $R_0$, with a coefficient of $-0.7$; other work, however, suggests that $R_0$ is nondivergent.\textsuperscript{11-14} We note that in all previous studies, the Hall effect was measured by applying a variable magnetic field, which has the unavoidable consequence of altering the conductivity, even in nominally nonmagnetic systems. This affects the occupation and hence the interactions of the highly correlated states especially near the MIT, possibly leading to the differing results discussed above. In the present work, we use a rotating magnetic field and take advantage of the large negative magnetoresistance of these alloys to measure the Hall effect at a constant magnetic field. By this means, we are able to measure $R_0$ and conductivity simultaneously and continuously through the MIT. It is the goal of this study to determine if $R_0$ is critical in the presence of both e-e interactions and localization, and to draw conclusions about the dependence of the carrier concentration (as determined by the Hall effect) on the tuning parameter driving the MIT.

The three-dimensional system $a$-Gd$_x$Si$_{1-x}$ can be reversibly tuned through the MIT by application of a magnetic field $H$.\textsuperscript{15} In this system, we have measured transport conductivity,\textsuperscript{12,16} density of states,\textsuperscript{5} magnetization,\textsuperscript{17} optical properties,\textsuperscript{18} and specific heat\textsuperscript{19} in the critical regime. The transport conductivity and the density of states at fixed $H$ on the metallic side of the MIT were shown to vary as $T^{7/2}$ and $E^{1/2}$, respectively, the same dependence as for nonmagnetic systems,\textsuperscript{1} with $T=0$ and $E=0$ offsets that depend on $H$. To study the dependence of the mobile carrier concentration $n$ on the magnetic field $H$, we measure the Hall coefficient for fixed values of $H$ at which the system is well into the metallic state to fixed values of $H$ at which the system is deep into the quantum critical regime. Since the goal is to study the $n$ dependence on $H$, we cannot use the conventional Hall geometry to determine $R_0$. Instead, as described in more detail below, we work at constant magnitude of $H$ (hence constant $n$ and magnetization $M$) and vary the angle between $H$ and the sample plane to obtain $R_0$. Because of the magnetic nature of the dopant atom Gd ($S=7/2$), we must also consider the contribution of the anomalous Hall effect to this measurement. For reasons to be discussed below in the context of the data obtained in this experiment, we think that the anomalous contribution is significantly smaller than the ordinary Hall effect near the MIT.

There is significant debate as to what constitutes the critical regime. In three dimensions, the Ioffe-Regel\textsuperscript{20} limit ($k_F l = 1$; where $k_F$ is the Fermi wave vector, $l$ is the mean free path) results in a conductivity, $\sigma = ne^2/(\hbar k_F^2)$ ($n$ is the electron concentration, $e$ is the electron charge, and $\hbar$ is Planck’s constant), below which a classical Fermi-liquid description fails to make sense. This conductivity is material dependant (due to $n/k_F^2 \propto n^{1/2}$) and may be considered a phe-
nomenologial upper limit for the conductivity $\sigma$ of the critical regime. The MIT in Gd$_{x}$Si$_{1-x}$ occurs for $x \approx 0.14$, with an electron concentration that is orders of magnitude larger than that found in crystalline doped semiconductor systems such as Si:P (for Gd$_{x}$Si$_{1-x}$, this electron concentration may be estimated as between $4 \times 10^{20}$ cm$^{-3}$ from optical absorption measurements and $2 \times 10^{22}$ cm$^{-3}$ from the Rutherford backscattering spectroscopy measured Gd concentration, assuming three donated electrons per Gd). Consequently, the Fermi temperature and the Ioffe-Regel limit on the conductivity in Gd$_{x}$Si$_{1-x}$ is significantly enhanced; taking $n \approx 10^{22}$ cm$^{-3}$ leads to a conductivity at the Ioffe-Regel limit ($\sigma \approx 500$ $\Omega^{-1}$ cm$^{-1}$), about 40 times larger than the equivalent value in Si:P, and allows us to experimentally probe the MIT deeper into the critical regime. Moreover, a high $E_F$ allows for a valid probing of the critical regime at a higher temperature, since the effective temperature $T/E_F$ is substantially reduced compared with the previous studies. In the present study, data have been acquired at $T = 400$ mK (ten times lower than previous data on amorphous systems). Temperatures 10-100 times smaller would be needed to obtain similar effective temperatures for crystalline doped semiconductors.

Measurements were made using Hall bars as shown in Fig. 1(a). These were fabricated using a lift-off technique. A film of $\approx 200$ nm of Cu was deposited on a SiN-covered Si surface, lithographically patterned, and etched into an inverse Hall bar pattern. Onto the resulting inverted Cu pattern $\approx 100$ nm of $\alpha$-Gd$_{x}$Si$_{1-x}$ was deposited, with $x \approx 0.14$, using electron-beam coevaporation of Gd and Si in an ultrahigh vacuum deposition system. The remaining Cu was then etched in a FeCl$_3$ solution, leaving the desired $\alpha$-Gd$_{x}$Si$_{1-x}$ Hall bar. The Hall bars have a current carrying strip and three pairs of voltage probes at three different locations along the strip, allowing for simultaneous measurements of the longitudinal ($R_{xx} = V_{xx}/I$) and the transverse ($R_{xy} = V_{xy}/I$) resistance. All resistances are measured using a low-frequency ($f \approx 30$ Hz) ac method.

Samples were measured using a $^3$He cryostat at $T = 400$ mK in applied magnetic fields $H$ to 33 T at the National High Magnetic Field Lab in Tallahassee, Florida. In order to measure $R_0$ in a system, where there is a magnetore-
by the vertical lines) yield \( R_{xy,odd} = H \sin \phi \).

To determine the central result, the Hall coefficient \( R_0 \), \( R_{xy,odd} \) determined in a manner described in the preceding paragraph, is plotted versus \( H \) for various \( H \) in Fig. 4. The Hall coefficient \( R_0 \) is related to the slope \( \Delta R_{xy,odd}/\Delta H_\perp \) according to \( R_0 = \tau \Delta R_{xy,odd}/\Delta H_\perp \), where \( \tau = 100 \) nm is the sample thickness. As \( H \) is reduced and the material approaches the insulating regime, we observe a concomitant increase in \( R_0 \).

In a magnetic metallic system, the Hall resistivity is expressed as

\[
R_{xy,odd} = R_0 H_\perp + R_S M_\perp,
\]

where \( M_\perp \) is the perpendicular component of magnetization. The first term describes the ordinary Hall effect with \( R_0 = (ne)^{-1} \) (\( e \) is the electron charge). The second term is the anomalous Hall effect, a result of interactions with magnetic moments, with the anomalous Hall coefficient \( R_S \). \(^{22}\) Since there is no magnetic anisotropy in these samples, \( M_\perp = M \sin \phi = (M/H) \times H_\perp \sin \phi = (M/H) \times H_\perp \), so Eq. (1) can be rewritten as

\[
R_{xy,odd} = (1/ne + R_S M/H) H_\perp.
\]  

At a fixed value of total field, \( H \), \( n \), \( R_S \), \( M \), and hence \( M/H \) are constant. Thus, both terms give a linear dependence on \( H_\perp \) for data taken in constant \( H \), as seen in Fig. 4. The dependence of \( R_0 \) (\( \approx 1/n \)) and \( R_S \) on field \( H \) is the issue to be determined.

Figure 5 shows the measured inverse Hall coefficient in units of \( (e \Delta R_{xy,odd}/\Delta H_\perp)^{-1} \) versus \( H \) for 100 kOe < \( H \) < 330 kOe, as well as the inverse of \( R_{xy,even}(H) = 1/2[R_{xy}(H) + R_{xy}(-H)] \), which as previously discussed is proportional to the longitudinal magnetoresistance \( R_{xx}(H) \). Both sets of data are linear and go to zero (\( R_{xy,odd} \) and \( R_{xy,even} \) divergent) for \( H = 100 \) kOe.

\[
R_{xy,odd} = (1/ne + R_S M/H) H_\perp.
\]
mobile carriers, \( n \), on the applied magnetic field \( H \), the driving parameter of the transition, with values as shown in Fig. 5.

We now argue that these results are dominated by the ordinary Hall effect and not by the anomalous Hall effect. For metallic systems, where \( n \) is large, the anomalous Hall coefficient dominates, as experimentally observed by Gambino and McGuire for \( \alpha\text{-Gd}_2\text{Ge}_1-x \) alloys.\(^{23} \) They found \( R_S \approx 5 \times 10^{-10} \text{ cm/G} \) (corresponding to \( n \approx 2.8 \times 10^{22} \text{ cm}^{-3} \)) and \( R_0 \approx -2.2 \times 10^{-12} \text{ cm/G} \) for \( x = 0.3 \). (Note: while still well on the metallic side of the MIT, i.e., the ordinary Hall coefficient is \( \approx 6\% \) of the anomalous Hall coefficient. For concentrated conventional magnetic metals, the anomalous Hall effect is even more dominant, typically a factor of \( 10^{10} \) larger than the ordinary Hall effect \( (R_S M / R_0 H \approx 10^{-1}) \). In the present experiment, however, two factors drastically reduce this ratio: both \( n \) and \( M \) are greatly decreased from the concentrated state. In the limit of \( n \to 0 \), \( R_0 \to \infty \). The dependence of \( R_S \) on \( n \) is not clear; assuming the anomalous Hall effect is dominated by skew scattering, as is likely,\(^{24} \) \( R_S \propto n^{2/3} \) in a free-electron picture, hence \( R_S \) is not divergent. Magnetization \( M \) vs \( H \) has been measured on these alloys to 250 kOe, using the magneto-optic Kerr effect.\(^{25} \) Using this data, and an assumption of \( R_S \propto n^{2/3} \), we have the ordinary Hall effect dominating the anomalous Hall effect by several orders of magnitude for the range of data shown. Furthermore, it would be difficult to conceive of a model in which \( (R_S M / H)^{-1} \) was linear in \( H \) near the MIT (i.e., \( 1/R_S M \) constant in \( H \)), requiring a peculiar cancellation of the separate strong dependences of \( M \) and \( R_S \) on \( H \). We therefore conclude that the natural and self-consistent solution is that at the MIT the ordinary Hall effect \( R_0 \) dominates, leading to the conclusion that the number of mobile carriers \( (n) \) depends linearly on the driving parameter in the quantum critical regime, \( (H-H_C) \).

In conclusion, while the precise conditions for an experimental realization of a system in the critical regime of a quantum critical transition are under debate,\(^{5} \) we have investigated a tunable system with conductivities up to two orders of magnitude below the Ioffe-Regel limit.\(^{20} \) In this system, we have measured transport,\(^{15,16} \) tunneling,\(^{5} \) magnetization,\(^{17} \) optical conductivity,\(^{18} \) and specific heat.\(^{19} \) Adding to this body of information, we have shown here that the Hall coefficient \( R_0 \) is a critical quantity, with critical exponent \( -1 \). We would like to thank Barry Zink and Geoffrey Marder for technical assistance and valuable discussions, the NSF and the AFOSR for financial support, and the NHMFL for financial and technical assistance.

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