

## 1/f noise in carbon nanotubes

Philip G. Collins, M. S. Fuhrer, and A. Zettl<sup>a)</sup>

*Department of Physics, University of California at Berkeley, and Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720*

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The electrical noise characteristics of single-walled carbon nanotubes have been investigated. For all three cases of individual isolated nanotubes, thin films of interconnected nanotubes, and bulk nanotube mats, anomalously large bias-dependent  $1/f$  noise is found. The noise magnitude greatly exceeds that commonly observed in metal films, carbon resistors, or even carbon fibers with comparable resistances. A single empirical expression describes the noise for all nanotube samples, suggesting a common noise-generating mechanism proportional only to the number of nanotubes in the conductor. We consider likely sources of the fluctuations, and consequences for electronic applications of nanotubes if the excessive noise cannot be suppressed. © 2000 American Institute of Physics. [S0003-6951(00)02701-7]

Carbon nanotubes<sup>1</sup> possess many novel properties that make them potential candidates for nanoscale electronics applications. The nanotubes may themselves serve as passive or active molecular-scale nonlinear devices,<sup>2-5</sup> or they may be useful as high-conductivity interconnects in a “mixed” architecture. One advantage afforded by carbon nanotubes is their presumed immunity to excess electrical noise. Excess noise, by which is meant noise above the unavoidable thermal Nyquist level, is a recognized barrier to practical, nanometer scale devices since it usually increases dramatically as dimensions shrink.<sup>6</sup> Both equilibrium and nonequilibrium fluctuations of a particular atom’s location, for example, gain importance as conduction paths are reduced to atomic dimensions. Carbon nanotubes, being covalently bonded metallic wires, might be less susceptible to such fluctuations. Furthermore, the strong carbon-carbon bonds which form the nanotube should not be subject to electromigration or defect propagation, two of the most important noise mechanisms in standard metal films and wires.<sup>7,8</sup>

We report here that, contrary to such expectations, metallic carbon nanotube conductors exhibit unexpectedly large electrical noise. Our measurements extend preliminary findings of excessive noise in a single nanotube device.<sup>9</sup> We have studied the room temperature noise characteristics of single-walled carbon nanotubes (SWNTs) in different configurations, ranging from isolated individual tubes to two-dimensional (2D) “films” and 3D “mats” of randomly interconnected nanotube assemblies. We find that all SWNT samples, irrespective of the contact electrode or tube connectivity configuration, display similar excessive  $1/f$  noise which cannot be explained within the idealized context of covalently bonded metallic wires.

The experiments were performed on heat-treated, single-walled carbon nanotubes grown by a laser ablation technique.<sup>10</sup> As grown, the bulk SWNT material formed a highly conductive, felt-like mat which was easily contacted for four-probe electrical characterization. These samples, which we denote as 3D, consisted of no less than  $10^6$

SWNTs, with a correspondingly high number of nanotube-nanotube junctions. The bulk SWNT material was also ultrasonically dispersed in dichloroethane and then deposited as thin films.<sup>11</sup> Using low-density solutions and closely-spaced electrodes, small bundles of 1–10 aligned SWNTs were investigated, albeit in a two-probe configuration. By increasing the electrode spacing, samples incorporating junctions were also measured. Control of the spacing and of the film thickness allowed a wide range of film morphologies to be studied. Throughout this letter, we will use the naming convention of 1D, 2D, and 3D to describe the different types of samples, with 1D reserved for samples which were determined by atomic force microscopy (AFM) to incorporate no more than one SWNT (or one small diameter SWNT bundle) between two electrodes.

The noise characterization was performed by biasing a sample at various dc levels and measuring the spectral density of low frequency fluctuations with a spectrum analyzer (HP3582A). In some cases, a lock-in amplifier (SR830) was employed as the detector. High resistance samples ( $R > 10 \text{ k}\Omega$ ) were voltage biased, and a transimpedance preamplifier (Ithaco 1211) was used to measure the sample current and current fluctuations. Low resistance samples ( $R \leq 1 \text{ k}\Omega$ ) were current biased, and voltage fluctuations were capacitatively coupled into a low noise transformer matched to a voltage preamplifier (PAR 1900 and 113). Although the voltage bias and current bias techniques measure different types of fluctuations, the results below are all uniformly presented in terms of effective voltage power fluctuations per Hz,  $S_v = \langle \Delta V^2 \rangle$ , with  $S_v = S_I R^2$ , with  $R$  the sample resistance. The noise measurements were restricted to bias in the linear resistance regime. Both two-probe and four-probe measurements were made on some of the 2D and 3D samples. Direct comparisons indicated that contact resistances to the SWNTs had no observable effect on the excess noise. Although highly suggestive, these results do not definitively rule out the effects of contact noise in the 1D samples.

Figure 1 depicts the noise power at different bias currents measured across an isolated, single SWNT. At zero

<sup>a)</sup>Electronic mail: azettl@physics.berkeley.edu

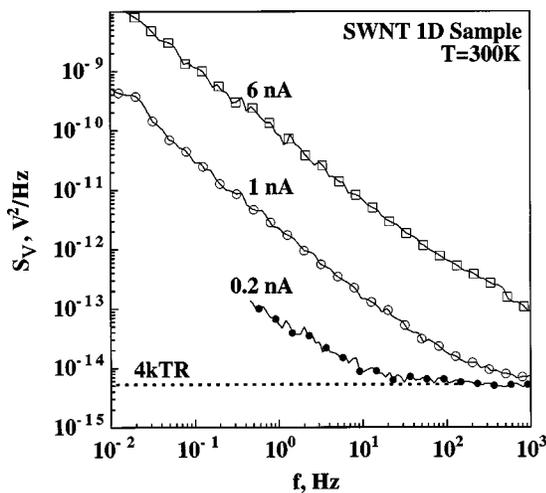


FIG. 1. Voltage noise power  $S_V$  vs frequency for a single SWNT, for three values of applied bias current. At low frequency, the noise greatly exceeds the thermal noise limit  $S_V=4kTR$ . The SWNT has a two probe resistance of 335 k $\Omega$ .

bias current, the noise was flat and agreed with the thermal Nyquist level  $S_V=4kTR$  (dashed line). At finite bias currents excess noise is observed. After subtracting the thermal baseline, the excess noise varies as  $1/f^\beta$ , with  $\beta=1.06\pm 0.02$  for all bias currents within the linear-response regime. Excess SWNT noise is  $1/f$ .

To determine the detailed bias dependence of the excess noise, a fixed frequency lock-in technique was employed. Figure 2 shows, for a single SWNT sample, the noise amplitude  $S_V$  as a function of bias current  $I$ , measured within a narrow frequency band centered on 10 Hz. The closed circles are the raw noise amplitude, while the open circles are  $S_V$  after subtracting the thermal baseline  $4kTR$ . For the cor-

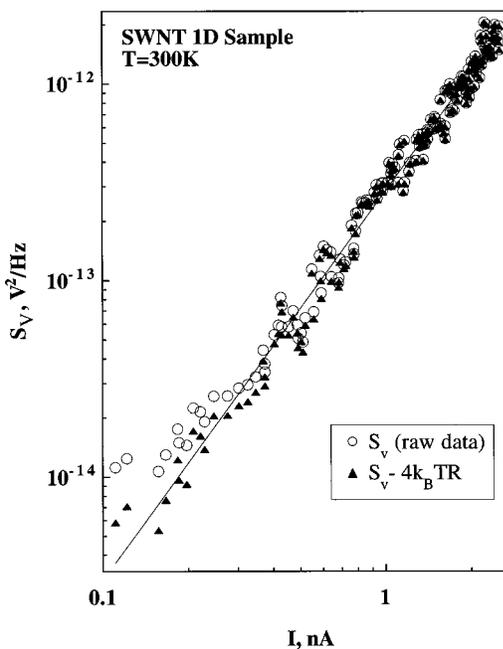


FIG. 2. Voltage noise power  $S_V$  as a function of dc bias current for a single SWNT. The closed circles represent the raw data, while for the open circles the bias-independent thermal noise has been subtracted. The excess noise fits an  $I^2$  behavior. The measurement is performed at 10 Hz with  $f=0.3$  Hz.

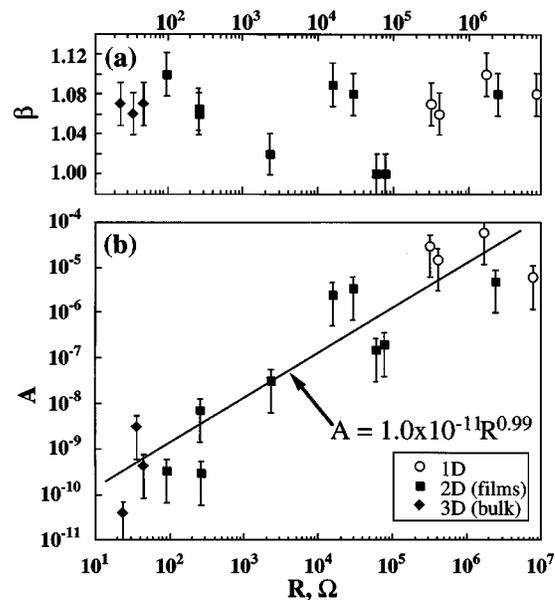


FIG. 3. Excess noise parameters for a wide range of nanotube-based conductors, as a function of sample resistance. The excess noise of each sample has the functional form  $S_V=AV^2/f^\beta$ . (a) Variation of  $\beta$  for the sample set. (b) Variation of  $A$  for the sample set. A least-squares power law fit suggests a direct proportionality between noise amplitude  $A$  and sample resistance  $R$ .

rected data, a bias current dependence of the form  $I^\nu$  is found, with  $\nu=1.99\pm 0.04$ .

Noise measurements similar to those described above for the single nanotube sample were repeated for a series of 1D, 2D, and 3D samples. Figure 3 summarizes the results. Different samples are distinguished by their dc resistance (horizontal axis), and different symbols correspond to different sample type (1D, 2D, 3D). Figure 3(a) shows the exponent  $\beta$  in the relation  $S_V\sim 1/f^\beta$ . For all samples  $\beta$  ranges between 1.00 and 1.10 with an average value of  $\beta=1.06$ , i.e., the excess noise is consistently  $1/f$ -like independent of whether the sample is 1D, 2D, or 3D. To characterize the absolute amplitude of the excess noise we express the voltage noise power as

$$S_V=AV^2/f^\beta. \tag{1}$$

This expression accounts well for the excess noise in all SWNT samples we have measured. Figure 3(b) shows the noise amplitude coefficient  $A$  plotted versus sample resistance, where we have used the average value  $\beta=1.06$  for all fits to Eq. (1). The solid line in Fig. 3(b) is a least-squares power law fit and indicates a direct proportionality between  $A$  and  $R$ :  $A=1.0\times 10^{-11}R$ . Therefore, while acknowledging the statistical limits of the data set, we may assign a universal behavior to excess noise in SWNT conductors: for all three morphologies, and for resistances spanning six orders of magnitude, the excess noise follows the behavior  $S_V=AV^2/f$ , with the dimensionless noise amplitude  $A$  now given by  $A/R=10^{-11}\Omega^{-1}$ .

It is illuminating to compare SWNT noise to that of other electronic conductors. Excess  $1/f$  noise is observed in a surprisingly wide variety of systems, and the functional form found here for SWNTs was already proposed three decades ago by Hooge<sup>6</sup> in a fluctuation model which unified a variety of available measurements. The  $V^2/f^\beta$  functional form has been theoretically considered for both linear<sup>7</sup> and nonlinear<sup>12</sup>

systems. Although some systems show deviations, most notably for “quantum”  $1/f$  noise,<sup>13</sup> the nearly universal behavior allows for straightforward comparisons between materials. For a characteristic device resistance  $R=100\ \Omega$ , SWNTs have a noise amplitude  $A=10^{-9}$ . High quality metal films tend to have values of  $A$  as small as  $10^{-19}$ , with values increasing to  $10^{-17}$  for thin films with strong grain boundary effects.<sup>14</sup> Metal films damaged by electromigration<sup>15</sup> or ion beam bombardment<sup>16</sup> show noise amplitudes approaching  $10^{-15}$ . Carbon composite resistors,<sup>17</sup> considered unsuitable for most low-noise circuitry, have excess noise amplitudes between  $10^{-15}$  and  $10^{-13}$  (for  $R\leq 1\ \text{k}\Omega$ ).<sup>14</sup> Carbon fibers<sup>18,19</sup> with resistances  $\leq 1\ \text{k}\Omega$  show similar noise magnitudes. Hence,  $1/f$  noise in SWNT conductors is four to ten orders of magnitude larger than that observed in more conventional conductors! Unless the excess noise can be somehow suppressed, this certainly calls into question the applicability of SWNTs for many low-noise electronic applications.

What is the origin of the excess noise in SWNTs? The expected noise immunity of a covalently bonded system is in competition with the increased relative importance of individual atomic fluctuations in nanometer-sized junctions. This size scaling is incorporated in Hooge’s empirical law, which expresses the excess noise magnitude as  $A=\alpha_H/N$ , where  $N$  is the number of atoms<sup>6</sup> or carriers<sup>20</sup> in the system and  $\alpha_H=0.002$  is a constant. Hooge’s law holds true for most bulk metallic systems, and even extends to the  $N=1$  case at the tip of a scanning tunneling microscope.<sup>21</sup> Estimating for the SWNT of Fig. 1  $N\sim 10^4$  atoms gives an estimate  $\alpha_H\approx 0.2$  for SWNTs, in sharp disagreement with Hooge’s law. In semiconductors,<sup>8</sup> and very small metal whiskers,<sup>22</sup> where surface and impurity fluctuations can dominate typical bulk effects,  $\alpha_H$  may be substantially larger than 0.002. The large value of  $\alpha_H$  observed for SWNT similarly suggests an important role played by surface fluctuations, a result not totally unexpected if one considers that every atom that constitutes a SWNT is a surface atom.

Our observation that  $A$  scales with  $R$  can also be illuminated by Hooge’s law. Combining  $A=10^{-11}R$  with Hooge’s law gives  $A=\alpha_H/N=10^{-11}R$ , whereby the number of carriers  $N$  is simply proportional to the sample conductance  $G=R^{-1}$ . From a qualitative point of view, this relationship merely reflects that both  $N$  and  $G$  depend on the number of parallel, conducting SWNTs in the sample.

Various extrinsic mechanisms might also be contributing to SWNT noise. Although our four-probe measurements seem to rule out dominant effects of noise at contacts, elec-

trical barriers at nanotube-nanotube junctions could produce fluctuations with a  $1/f$  power spectrum. Measurements on single SWNTs do not incorporate such junctions, but have only been accomplished in a two-probe configuration. Furthermore, the electronic effects of mobile adsorbents and other possible contaminants have not been ruled out. Given that large  $\alpha_H$  values suggest the importance of surface mechanisms, any adsorbates or intercalants which affect a SWNT electronically might also be expected to play a role in the generation of  $1/f$  noise.

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