Limits of Nanomechanical Resonators

K. Jensen*,† , H.B. Peng*,‡, and A. Zettl*,†,‡
*Department of Physics, University of California at Berkeley, Berkeley, CA 94720 U.S.A
†Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA, 94720 U.S.A
‡Center of Integrated Nanomechanical Systems, University of California at Berkeley, Berkeley, CA, 94720 U.S.A

Abstract—**We have constructed nanotube-based nanomechanical resonators designed to investigate the upper limits of nanomechanical resonator frequency and quality factor. Our ultra-high-frequency resonator relies on abacus-style inertial clamps of metal deposited along the length of the nanotube beam to shorten its effective length and thus raise its resonant frequency. Our high-Q nanoresonator design utilizes the unique telescoping property of multiwalled carbon nanotubes to obtain a pristine surface, which may help reduce dissipation.**

NEMS; resonator; high frequency; Q; dissipation

I. INTRODUCTION

Human-made mechanical resonators have been in development for thousands of years. Early applications included musical instruments and chronographs operating at millihertz to kilohertz frequencies, while more recent interest has turned to ultra-high frequency oscillators useful to wireless communications technology and single-molecule detection schemes. Resonators with exceptionally high quality factors, Q, have been used for exotic physics experiments such as an examination of the distinction between gravitational mass and inertial mass, the detection of gravity waves, and the behavior of low-occupation number quantum oscillators. The present "holy grail" of mechanical resonators is the development of RF and microwave oscillators with uncompromised Q. The trend has been towards small, stiff, and low mass systems, from microelectromechanical (MEMS) towards nanoelectromechanical (NEMS) systems. Although some impressive progress has been made with "conventional" stiff MEMS materials such as silicon or silicon nitride, these materials often have less than ideal surface structures or chemistries. Many studies, even on macroscopic systems, have demonstrated a key role played by surface interactions on loss mechanisms. As systems scale down in size, surface-to-volume ratios invariably increase, making surface dissipation modes ever more critical.

In this report we investigate some fundamental issues of frequency and quality factor of NEMS oscillators. Prime materials that might overcome surface problems include nanotubes formed from lightweight elements forming structures with ultra-strong sp^2 bonds, few topological defects, and low surface reactivities. We discuss two different implementations of nanotube-based tunable resonators and evaluate their performance. We find that nanotube-based resonators with ultra-high frequency response (high MHz to >1GHz) can be constructed in a relatively straightforward manner and that ultra-high Q's may be obtainable.

II. ULTRA-HIGH FREQUENCY NANOMECHANICAL RESONATORS

For a rigid elastic beam, resonant frequency of the transverse flexural mode is inversely proportional to the square of the beam length. Thus, to obtain high frequency mechanical resonators, it is necessary to shrink the size of the resonator. This explains the current popularity of nanomechanical resonators. Yet, creating a useful, high frequency mechanical resonator is no simple task. The challenge is two-fold: devices must be fabricated with nanoscale dimensions, and detection methods must have suitable sensitivity to the sub-nanoscale displacements. The previous high frequency record-holder for nanomechanical resonators was a doubly-clamped beam constructed from stiff 3C-SiC with a resonant frequency of 1.029 GHz. The resonance of this device was detected only at low temperatures (4.2 K) in ultrahigh vacuum (UHV) [1]. Because of their higher stiffness, carbon nanotubes (CNTs) provide a possibly better material to construct resonators. Moreover, previous research has demonstrated the electrical detection of their resonance, although the frequencies were below 200 MHz, and no resonance was detectable at atmospheric pressure [2-5].

We have constructed suspended CNT-based resonators with an ultra-high fundamental resonant frequency of ~4 GHz. The mechanical motion of the resonator was self-detected at room temperature in air at atmospheric pressure. [6,7] Our breakthrough results from two critical improvements. First, drive and detection methods with built-in mixing techniques were developed to detect the ultra-small mechanical displacement of the resonating beam.[5,6] Second, unconventional techniques were applied to construct nanoscale resonators beyond the reach of standard lithograpy. More specifically, we loaded the suspended carbon nanotubes abacus-style with inertial metal clamps yielding short effective beam lengths.[7] These nanotube-based "nano-abacus" devices functioning as self-detecting NEMS resonators, are capable of operating in ambient-pressure air at room temperature at microwave frequencies.

Figure 1 shows schematically the nano-abacus device configuration. A single-wall carbon nanotube (SWNT) is grown by chemical vapor deposition across a trench between metal source and drain electrodes. [6,7] A localized gate electrode lies at the bottom of the trench. After SWNT growth, metal beads are placed abacus-style on the CNTs by evaporating metal (such as indium) over the entire device. When driven mechanically, the CNT exhibits transverse-beam-like flexural modes. The crucial point is that shortened CNT

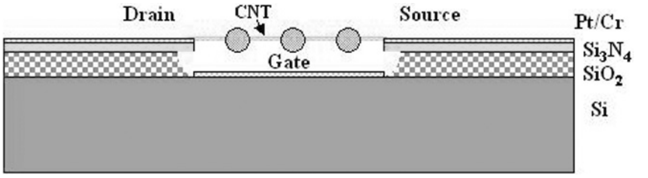

Figure 1. Schematic of nano-abacus high frequency resonator. A SWNT nanotube with metal inertial clamps is suspended between the drain and source. An RF signal from the gate actuates the resonator.

sections, between two adjacent indium beads, can serve as the resonating beam, and the short effective clamping length results in ultra-high resonance frequencies.

Figure 2 gives the electrical response at room temperature of a self-detecting nano-abacus oscillator. A high-sensitivity "2ω" mixing method is here used to drive and electrically detect the mechanical motion of the nano-abacus oscillator. [6] In brief, an actuation RF signal is applied to the gate at frequency ω and a carrier signal is applied to the drain at a different frequency 2ω-Δω. The drain-source current is monitored by a lock-in amplifier at the intermediate frequency Δω. The nanotube serves as a nonlinear mixer.

As seen from Fig.2, when the resonator is driven through resonance, sharp change occurs in both the amplitude and the phase of the measured AC electrical current. The resonance frequency of our device is ~1.32 GHz with a quality factor Q of ~50.

III. DISSIPATION IN NANOMECHANICAL RESONATORS

Besides frequency, a major figure of merit for any resonator is of course its quality factor. For nanomechanical resonators in particular, Q determines the sensitivity of nanomechanical mass sensors [8] and whether the resonator will be useful as an RF filter. Unfortunately, the quality factors of nanomechanical resonators, including nanotube-based resonators, have been rather disappointing, typically below 100 and only rarely exceeding 1000. There is hope that these low quality factors are not due to any fundamental limitation, and that they may be controlled through careful experimental techniques.

Low quality factors result from dissipation of a resonator's mechanical energy. There are numerous possible sources of dissipation in a nanomechanical resonator, which may broadly be classified as either intrinsic or extrinsic. Intrinsic sources of dissipation, such as phonon-phonon and phonon-electron interactions, result from properties of the resonating material, whereas extrinsic sources, such as gas friction, clamping loss, and surface loss, result from interactions with the environment. Obviously, little can be done to control dissipation from intrinsic sources other than careful choice of resonator material. Theoretical calculations have shown that these intrinsic sources of dissipation are small compared to the dissipation currently exhibited by nanomechanical resonators. Much, however, can be done to limit dissipation from the extrinsic sources. Below we review three dominant extrinsic sources of dissipation for nanomechanical systems and methods that can be used to eliminate them.

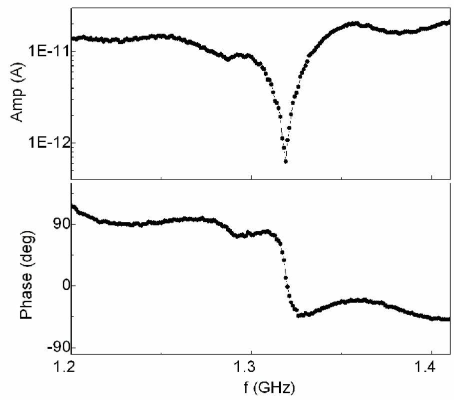

Figure 2. Typical resonance curves for our high frequency resonator.

A. *Gas friction*

At pressures above approximately 1 torr, viscous damping of a resonator by the surrounding gas is the dominant form of dissipation [9]. Here the energy is radiated as sound. Fortunately, it is easy to achieve lower pressures where viscous damping no longer dominates. At these lower pressures, where the mean free path of the gas molecules is much larger than the relevant sound wavelength, energy may still be dissipated through momentum transfer to individual molecules. In this case the dissipation is calculated to be:

$$Q_{gas}^{-1} = \frac{pA}{M_{eff}\,\omega v} \tag{1}$$

where p is the pressure, A is the surface area, M_{eff} is the effective mass of the resonator, ω is the resonator's angular frequency, and v is the thermal velocity of the gas. According to Eq. 1 and multiple experiments, gas friction is not a significant source of dissipation below ~10 mTorr. Our nanotube resonator experiments are conducted at 10^{-7} torr, and thus gas friction has little effect on their Q factors.

B. *Clamping loss*

Clamping loss refers to mechanical energy dissipated through a resonator's supports. Typically, this is theoretically modeled as elastic radiation of energy through the supports. There is still some contention as to the appropriate description of elastic clamping loss; though the most recent theoretical calculations predict a loss for a rectangular beam of:

$$Q_{clamping}^{-1} \propto \frac{wt^4}{l^5} \tag{2}$$

where w is the beam width, t is the beam thickness, l is the beam length, and the proportionality constant is dependent upon material properties [10]. Clearly, to reduce clamping loss, a beam with a high aspect ratio is desirable. However, according to calculations performed using Eq. 2, clamping loss should be negligible for current resonator designs, including our nanotube resonators with their extremely high aspect ratio.

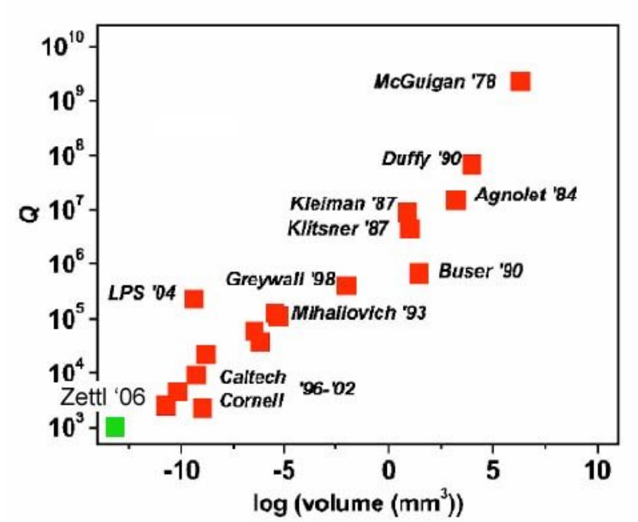

Figure 3. Trend in quality factors as a function of resonator volume over many orders of magnitude. (Plot modified from [9]).

Despite these model-based constraints, researchers have had some success increasing quality factors through creative clamping geometries[11]. Thus, it is clear that the theory behind clamping loss is not fully developed, and that this may still prove to have been a significant form of dissipation for existing NEMS resonators.

C. Surface loss

Surface losses are caused by adsorbed molecules, dangling or broken bonds, an amorphous oxide layer, or other metastable systems that occur at a resonator's surface. These systems absorb energy from the fundamental resonant mode and irreversibly transfer it other modes and thermal energy.

There is strong evidence for the importance of surface losses at the nanoscale. In particular, Mohanty et al., have noticed a disturbing trend in the quality factors of resonators as a function of resonator volume (Fig. 3). From the giant resonant bar gravitational wave antennas to the smallest nanotube resonator, as the resonator gets smaller so does its quality factor. Indeed, our own nanotube resonators, which have relatively high Q's when compared to other nanotube resonators [5], follow the trend comfortably, as indicated by the left-most datum in Fig. 3.

To explain how this trend supports the dominance of surface losses, consider the definition for the quality factor, $Q = 2\pi E_0 / \Delta E$, where E_0 is the energy initially stored in the resonator and ΔE is the loss of energy per cycle. For resonating beams, the energy of a resonator is stored in the elastic strain throughout its volume and thus is proportional to its volume, V. If we assume that energy is predominately dissipated at the surface, then we would expect that the energy lost per cycle would be proportional to the surface area S, and thus:

$$Q_{surface}^{-1} \propto S / V \propto L^{-1} \tag{3}$$

which describes the trend in Fig 3.

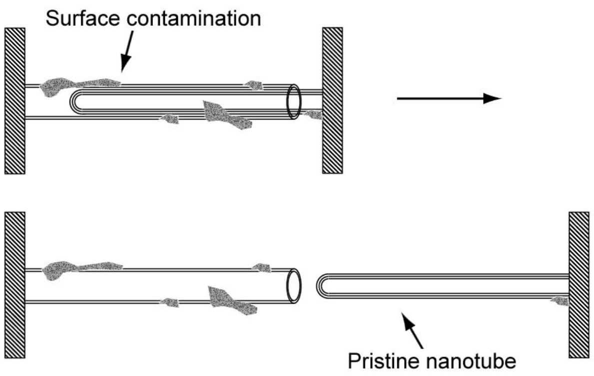

Figure 4. Method for obtaining a nanotube resonator with a contamination-free surface.

Fortunately, it may be possible to control surface losses through careful experimental techniques and the proper choice of resonator material. CNTs provide possibly the ideal material for constructing high-Q nanomechanical resonators. CNTs have smooth surfaces unlike the rough, irregular surfaces of micromachined silicon. Moreover, the crystal structure of carbon nanotubes naturally terminates at the surface, so there should be no dangling bonds.

Resonators constructed from telescoping MWNTs[12], where a core nanotube may slide within the atomically smooth casing of a shell nanotube, provide an excellent platform for study of the effects of surface contamination. This system allows the precise control of a resonator's surface area while leaving all other factors (clamps, diameter, mass, etc...) the same. Already, this technique has been used to provide more evidence for surface-area dependent dissipation [12]. Possibly more exciting though, by extracting the core of a MWNT, as shown schematically in Fig. 4, under UHV conditions, it may be possible to access a pristine MWNT whose surface has never been contaminated by exposure to air or other contaminating species. This would provide the ultimate test of the effects of surface contaminations and moreover, by eliminating what may be the dominant form of dissipation, could lead to ultra-high Q nanotube resonators.

D. Ultimate limits of Q

While Fig. 3 clearly shows the decrease in Q from macroscopic resonators to nanoscale resonators, there is some hope on the smaller, molecular end of the size scale that this trend is not fundamental. Microwave spectroscopy measurements of the mechanical vibrations of single molecules (O_2, H_2O) can have extremely narrow spectral widths at low pressures. In fact, for these molecular vibrations, Q is ultimately limited only by disturbances from vacuum fluctuations of the electromagnetic field. The "natural" line-width for these modes can be calculated to be on the order of 10^{17} [13].

More realistically, for nanomechanical resonators the quality factors will likely be limited by thermoelastic dissipation. In this mode of dissipation, strain in the resonator generates local temperature differences via the material's thermal-expansion coefficient. Heat then flows irreversibly along local temperature gradients leading to dissipation. However, even in this case, quality factors on the order of 10^{14} are still obtainable at low temperatures [14].

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