

A Carbon Nanotube-based NEMS Parametric Amplifier for Enhanced Radio Wave Detection and Electronic Signal Amplification

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Abstract. We propose a scheme for a parametric amplifier based on a single suspended carbon nanotube field-emitter. This novel electromechanical nanotube device acts as a phase-sensitive, variable-gain, band-pass-filtering amplifier for electronic signal processing and, at the same time, can operate as a variable-sensitivity, tuneable detector and transducer of radio frequency electromagnetic waves. The amplifier can exhibit infinite gain at pumping voltages much less than 10 Volts. Additionally, the amplifier's low overhead power consumption (10-1000 nW) make it exceptionally attractive for ultra-low-power applications.

1. Introduction

The phenomenon of parametric amplification, such as occurs in a child's swing, has been studied since the early 19th century^{1,2} and occurs in oscillating systems in which a parameter (*i.e.* resonance frequency, spring constant, degree of dissipation, *etc.*) is modulated at a sub-multiple of twice the resonance frequency, $2\omega_0/n$, to produce amplification in the response of the system. Parametric amplifiers have generated sustained interest because their operational noise can approach the quantum-limit^{3,4} and several implementations of phenomenon have been demonstrated in electronic systems using varactors⁵ and Josephson junctions⁶, optical systems using nonlinear materials⁷, and in electromechanical systems⁸. In this paper, we explore the parametric amplification of mechanical oscillations in a suspended carbon nanotube resonator. We show that by operating the device in the limit of strong coupling between mechanical vibrations and field emission tunnelling current, the device behaves as a tuneable, high-gain, phase-sensitive amplifier of AC current and voltage signals. Additionally, the same device can be used as a tuneable, highly-sensitive detector of radio frequency (RF) electromagnetic radiation or as an RF transmitter.

Suspended carbon nanotube field emitters have been explored as electron sources^{9, 10, 11} and Fowler-Nordheim diodes¹², and have shown exceptional promise as resonators¹³ in nanoelectromechanical systems (NEMS) with applications to radio wave detection¹⁴, atomic-resolution inertial mass sensing¹⁵, and self-sustained RF oscillators¹⁶. Apart from be able to actuate

carbon nanotube field-emission devices with electromagnetic fields, they offer a unique way to read out vibrational information because the field-emission current is strongly coupled to the position of the nanotube relative to the counter-electrode. This coupling occurs because the field emission current is dominated by the local field at the nanotube tip which can vary substantially as the tube position changes relative to the counter-electrode^{14, 16}. Therefore, the vibrational dynamics of the tube can be ascertained from a direct monitoring the field emission current.

We propose a scheme for parametric amplification using a single suspended carbon nanotube field emitter that can operate as a current or voltage amplifier or as an electromagnetic wave transducer, with a direct field emission current output signal. A three-terminal prototype of such a device is shown in Figure 1 (a); the lithographically fabricated device consists of a singly-clamped multi-walled carbon nanotube, a counter (drain) electrode, and a pumping electrode to generate parametric modulation. We begin by describing the nature of the parametric modulation in our system and then explore the device's theoretical performance as an amplifier and transducer.

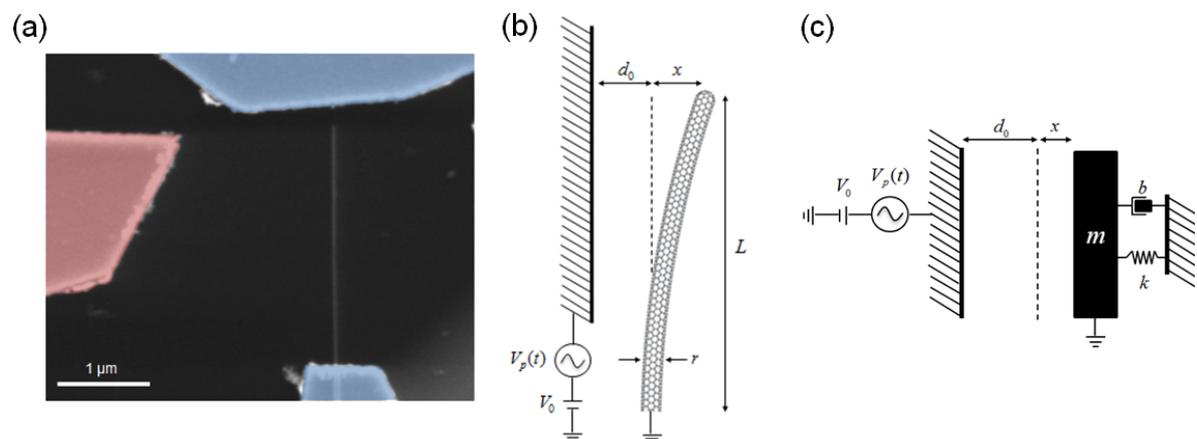


Figure 1: Prototype for a carbon nanotube parametric amplifier. (a) A false-color scanning electron micrograph of a lithographically-defined multiwalled carbon nanotube device. The source and drain electrodes (upper and lower in blue) maintain and measure the field emission current and the side pumping electrode (left in red) generates the modulation of the effective elastic spring constant of the nanotube. (b) A schematic of the device highlighting the interaction with the pump electrode and the basic electrical circuit wiring. (c) A schematic of the equivalent electromechanical system for the pump-nanotube interaction shown in (b). The pump electrode-nanotube interaction is modelled as a position variable capacitor of mass m connected to a rigid support by a spring of spring constant k and dissipation b .

2. Results and Discussion

We model our singly-clamped carbon nanotube device as a capacitively-coupled, driven-damped simple harmonic oscillator (Figure 1 (b) and (c)). The electromechanical coupling in the presence of a time-varying potential $V(t)$, leads to a Hooke's Law force term with a time-dependent spring constant in the total electrostatic force, $F_E = -\frac{1}{2} \frac{dC}{dx} V^2(t)$, which, in turn, is responsible for the amplified response of the nanotube vibrations in the presence of an external driving force.

The form of the modulated spring constant is obtained from the capacitance of the nanotube, which we approximate with the classical expression for the capacitance of a metallic cylinder of radius r and length L a distance $d = d_0 + x$ from a conducting plane¹⁷,

$$C(x) = \frac{2\pi\epsilon_0}{\log\left(\frac{4d_0(1+x/d_0)}{r}\right)} L, \quad (1)$$

where x is the deflection of the nanotube tip, d_0 is the equilibrium tip distance in the absence of any forces, and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$ is the vacuum permittivity. Although we are using x as the tip deflection, Eq. (1) agrees well (within 18 %) with finite-element simulations performed on a nanotube bending under the appropriate electrostatic load¹⁸. Some error cancellation is expected since we have neglected the capacitive contribution of the highly charged nanotube tip¹⁶. Preserving the first three terms of the Taylor expansion of Eq. (1) for $x < d_0$ and applying a voltage $V(t) = V_0 + V_p \sin(2\omega_0 t)$ to the pump electrode we obtain

$$F_E = -\frac{1}{2} \left(\frac{dC(0)}{dx} + \frac{d^2C(0)}{dx^2} x \right) (V_0^2 + 2V_0V_p \sin(2\omega_0 t) + V_p^2 \sin^2(2\omega_0 t)) \quad (2)$$

Retaining only the terms involving products of x with V_0^2 or $\sin(2\omega_0 t)$ (neglected terms result in static deflection or produce off-resonant driving forces), and including dissipative, elastic, and driving force terms, the equation of motion of the nanotube becomes

$$m \frac{d^2 x}{dt^2} + m \frac{\omega_0}{Q} \frac{dx}{dt} + [k + \Delta k \sin(2\omega_0 t)] x = F \cos(\omega_0 t + \phi) \quad (3)$$

where m is the nanotube mass, Q is the resonator quality factor, $\omega_0 = \omega_0(k)$ is the mechanical resonance frequency, $k = k_0 + \frac{1}{2} \frac{d^2C(0)}{dx^2} V_0^2$, and $\Delta k = \frac{d^2C(0)}{dx^2} V_0 V_p$.

We use a standard result¹⁹ from elastic beam theory for the spring constant, $k_0 = \frac{3\pi E r^4}{4L^3}$, with a Young's modulus $E \approx 1 \text{ TPa}$, and use Eq. (1) to obtain

$$\frac{d^2C(0)}{dx^2} = \frac{\pi\epsilon_0 L}{d_0^2} \frac{\left(2 + \text{Ln}\left(\frac{4d_0}{r}\right)\right)}{\text{Ln}^3\left(\frac{4d_0}{r}\right)}. \quad (4)$$

Eq. (3) is in the form of the driven Mathieu equation, and we therefore expect the amplitude-frequency response of the nanotube to exhibit amplification governed by the relative strength of the modulated term $\Delta k(t)$. The solution for amplitude response of Eq. (3) in the limit $Q \gg 1$ is

$A = F \frac{Q\omega_0}{k} G(\phi)$ ^{8,20}, where the phase-sensitive gain is given by

$$G(V_0, V_p, \phi) = \left[\frac{\cos^2 \phi}{(1 + Q\Delta k / 2k)^2} + \frac{\sin^2 \phi}{(1 - Q\Delta k / 2k)^2} \right]^{1/2} \quad (5)$$

The gain of our amplifier is therefore specified by V_p , V_0 , and ϕ . It is a well-known fact that tuning the phase to zero in a parametric amplifier leads to de-amplification, while setting $\phi = \pi/2$ yields maximum amplification. Figure 2 shows $G(\phi)$ for $\phi = \pi/2$ as a function of V_p for device geometries similar that shown in Figure 1 (a) and to those from our previous experiments^{14, 15, 16} (e.g. nanotubes of length $L = 1 \mu\text{m}$, radius $r = 5 \text{ nm}$, and $Q = 500$). For fixed values of V_0 , the gain is rather modest ($G \sim 10$) for low V_p but quickly increases to values approaching infinity for finite V_p . This divergence marks the threshold for instability and parametric oscillations⁸.

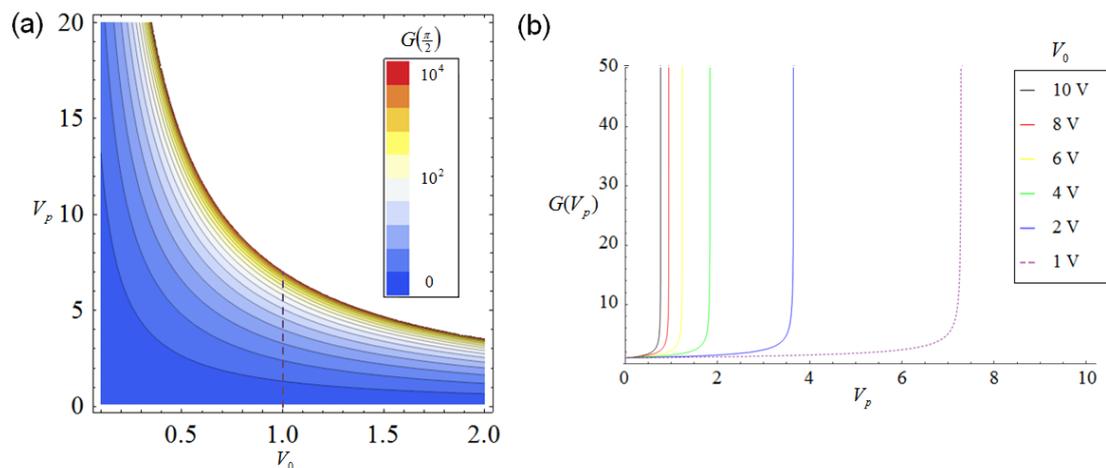


Figure 2: Amplifier Gain: (a) Gain of amplifier as a function of V_0 and V_p . The upper white portion of the graph is a region of resonator instability. (b) Shows gain as a function of V_p for several fixed values of V_0 . The dashed line at $V_0 = 1 \text{ V}$ in (a) corresponds to the rightmost dashed line in this plot and aids in visualizing the rapid change in gain near a critical value (about 7.25 V for the dashed curve) of V_p . In all the curves, the gain is near unity for lower values of V_p and then quickly diverges to infinity. In all plots, $L = d_0 = 1 \mu\text{m}$, $r = 5 \text{ nm}$, $Q = 500$, and $\phi = \pi/2$.

The carbon nanotube NEMS device discussed so far will amplify or detect the Fourier components of *any* signal that are both centered within the bandwidth ($\Delta\omega_0 = \omega_0/Q$) of the nanotube's vibrational resonance frequency and capable of exciting the nanotube's vibrational modes into resonance. In this way, our device functions as a tunable band pass filter while simultaneously serving as an amplifier and a detector (Figure 3).

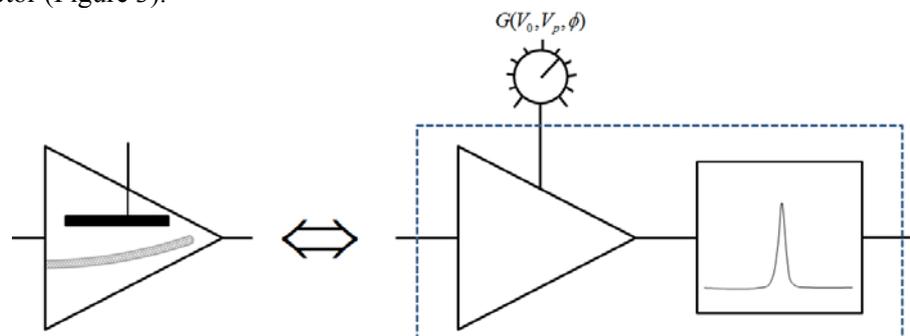


Figure 3: Schematic and Circuit Symbol of Carbon Nanotube-based NEMS Parametric Amplifier. A proposed circuit (left) symbol for our

nanoelectromechanical carbon nanotube paramp which serves as a band-pass-filtered, variable-gain, phase-sensitive amplifier (right).

The bandwidth of the filtering process will be on the order of 100 kHz for a 100 MHz nanotube resonator. While operating in amplification or detection mode, the gain/sensitivity is controlled by V_p , V_0 , and ϕ . The resonance frequency, through its dependence on the spring constant k , can be tuned by adjusting V_0 . Note that this capacitive frequency tunability is in addition to the electrostatic tensioning of the nanotube with V_{NT} ^{13,14}. Chip-based devices, as shown in Figure 1 (a), can be readily fabricated and engineered to operate in the 100 kHz – 1 GHz frequency band.

AC current and voltage signals in the appropriate band will be amplified. For instance, applying an AC voltage to the pump probe or the nanotube electrode at ω_0 will drive the nanotube into resonance (with an amplified response due to Δk) resulting in an AC current through the source-drain circuit path. Similarly, a current driven through the nanotube at ω_0 will modulate the static charge on the tube and, because of V_0 , excite oscillations which are in turn amplified by the parametric pump.

Detection of RF electromagnetic radiation by a suspended, field-emitting carbon nanotube has been reported in previous work¹⁴. Signal transduction occurs because the RF wave drives the nanotube into oscillation resulting, once again, in a current signal at the output of the device. Our parametrically modulated system adds improvements to the nanotube radio because we can enhance the sensitivity to incoming RF photons by several orders of magnitude, possibly by as much as a factor of 10^5 - 10^8 . Furthermore, a high-frequency device operated in the limit of high gain, in order to induce parametric self-oscillations¹⁶, could conceivably be used to generate detectable RF dipole radiation.

3. Conclusion

We have described a novel electromechanical nanotube device that acts as a phase-sensitive, variable-gain filtering amplifier of electronic signals and, at the same time, can operate as a variable-sensitivity detector/transducer of RF electromagnetic waves. The amplifier can exhibit infinite gain at pumping voltages much less than 10 Volts. Additionally, the amplifier's overhead power consumption will be on the order of 10-1000 nW, making it an exceptionally low-power amplifier. Ultimate gain limitations of the described nanotube amplifier will be governed by nonlinear effects and the maximum achievable amplitude of the nanotube resonator. Also, the operational range for incoming signals will be limited by the current capacity of the nanotube, which is typically $\sim 10 - 100 \mu\text{A}$. We expect this highly-tunable carbon nanotube paramp will find many useful applications in electronics, as a filter and an amplifier, in wireless communications, as a low-power RF receiver and transmitter, and in fundamental science as a versatile NEMS device for studies in condensed matter and as a highly-sensitive RF photon detector in optics.

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