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DYNAMICS OF CHARGE DENSITY WAVE CONDUCTORS: BROKEN COHERENCE, CHAOS, AND NOISY PRECURSORS†

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Chaotic dynamics resulting from a macroscopic breaking of charge density wave coherence are investigated in $NbSe_3$. The ac-dc electric field induced period doubling bifurcation is associated with a new noisy precursor effect, the virtual Hopf phenomenon.

1. INTRODUCTION

A number of recent experiments on charge density wave (CDW) conductors have firmly established the existance of chaotic CDW response for ac, dc, and combined ac + dc electric drive fields. In $(TaSe_4)_2I$, for example, chaotic ac conductivity results if the crystal is series coupled to an inertia-providing inductance and driven with an ac voltage.[1] In $NbSe_3$ and Fe_xNbSe_3, highly complex behavior has been observed for dc drive fields alone in a negative differential resistance region.[2] The response there is associated with apparently random hopping between quasi-stable and distinct current-carrying states, leading to intermittency and strong 1/f noise. Perhaps the most dramatic example of chaos in CDW systems occurs for combined ac and dc drive fields, where period doubling bifurcations are observed on mode-locked Shapiro steps.[3] We here consider some aspects of this (period doubling) route to chaos.

2. BROKEN COHERENCE AND CHAOS

As has been appreciated for some time, the depinned (i.e. sliding) CDW condensate displays a high degree of dynamic phase coherence.[4] The narrow band noise frequency, which identifies the intrinsic limit cycle of the system, is in $NbSe_3$ extremely sharp, and well defined in terms of CDW phase velocity. Mixing of external rf radiation with the internal oscillations results in Shapiro step interference, an effective electronic locking of the CDW phase mode. Despite a rich spectrum of harmonic[4] and subharmonic[5] interference structure, a CDW crystal with smooth dc depinning behavior (i.e. a "non-switching" sample) will *not* display chaotic response, even if driven very hard. This means that transitions to chaos, as might be expected from an overlapping of resonances as in the sine circle map,[6] are not realized in such macroscopically coherent crystals.

Chaotic response in $NbSe_3$ (and most probably in related materials such as TaS_3, $K_{0.3}MoO_3$, etc.) results when the macroscopic phase coherence is broken, in particular by the existance of bulk phase slip centers. Bulk phase slip centers (not to be confused with phase slip vortices which may appear at conductive contacts[7]) lead directly to switching,[8] and switching leads to chaos.[2,3]

Although switching and chaos can be, to a fair degree, described by *underdamped* classical

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dynamics (for example, by simply assuming an inertial term in a damped pendulum-like equation[3]), a more realistic approach treats the dynamics of the CDW in the crystal as *overdamped but coupled to amplitude dynamics at the phase slip interface*. The coupled dynamics dictate the overall chaotic response. A simple model of phase slip centers has been developed which explicitly incorporates the necessary coupling between a polarizable CDW condensate and a phase slip interface.[9] At the interface, the CDW amplitude becomes a dynamical variable. The model leads to hysteretic switching behavior in the I-V characteristics,[9] and mode locking, period doubling, and chaos for combined ac and dc drive fields,[10] while still retaining the (experimentally observed) overdamped form of the low field ac conductivity.

3. PERIOD DOUBLING BIFURCATIONS AND NOISY PRECURSORS

We now consider explicitly the period doubling route to chaos. Fig. 1 shows a period doubling bifurcation sequence in $NbSe_3$, induced by steadily increasing the dc bias field with fixed external ac amplitude and frequency. Period 1,2,4, and chaotic responses are clearly observed. The period doubling bifurcation induced by the variation of a single parameter, in this case the dc bias field, is an example of loss of stability in the periodic orbit. This particular instability is one of four so-called co-dimension one bifurcations (the others being saddle node, transcritical, and Hopf bifurcations). Interestingly, there exist well-defined scenarios for the dynamical system response *before* the actual instability (e.g. period doubling) occurs. Each class of co-dimension one bifurcations displays, in the presence of some external noise, unique "noisy precursor" effects,[11] which depend only on the bifurcation class, not on the details of the chaotic system. The precursor effects are manifestations of the extreme sensitivity of the system to noise near the bifurcation instability. For a period doubling instability, the expected precursor effect results in a power spectrum with rounded noise "bumps" centered at one half the fundamental frequency. After the bifurcation takes place, the half frequency spike continues to grow and eventually emerges from the precursor bump; the bump itself diminishes. Figures 2a and 2b show the expected power spectrum in the vicinity of a period doubling bifurcation.[11] Remnant noisy precursor features analogous to those shown in Fig. 2b occur in $NbSe_3$. For example, remnant precursor bump structure is apparent in Fig. 1, most notably in the period 4 regime, (iii). Of importance is the lack of any bump structure on the peak corresponding to the external drive frequency, consistent with theory.

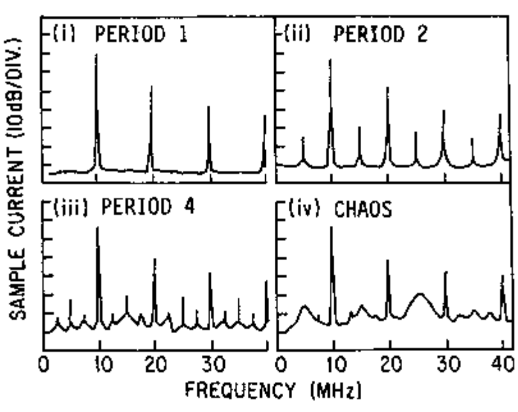

FIGURE 1
Period doubling route to chaos in $NbSe_3$. Ac field amplitude is 21 mV at 10 MHz, dc field amplitude is (i) 25 mV, (ii) 25.1 mV, (iii) 25.2 mV, (iv) 25.5 mV

A particularly interesting noisy precursor effect associated with period doubling bifurcations (but heretofore unobserved experimentally) is the virtual Hopf phenomenon.[12] This phenomenon results in a continuous change (again as a single paramater, e.g. dc bias, is varied) from the noisy precursor expected for a Hopf bifurcation (see below) to the noisy precursor of the period doubling bifurcation. Figures 3a-d show

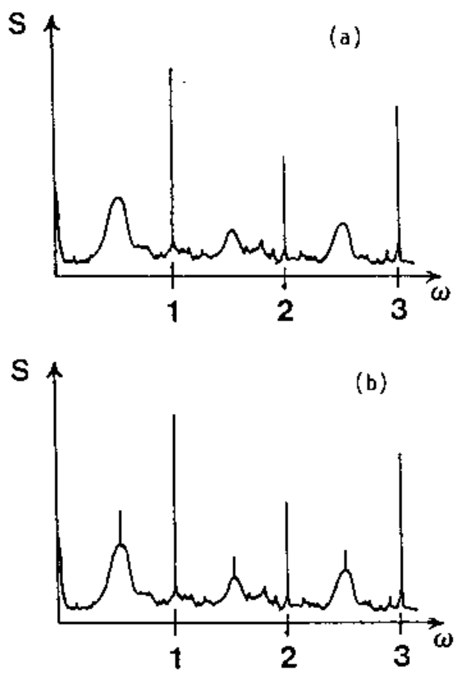

FIGURE 2
(a) Expected noisy precursor power spectrum just before a period doubling instability. (b) Power spectrum following period doubling bifurcation. From ref. 11

the predicted sequence of events. The power spectrum of Fig. 3a is that expected[11] just prior to a Hopf bifurcation, where additional frequency peaks form at frequencies ω' and $\omega_0 - \omega'$. Figure 3d illustrates the bump structure at $\omega_0/2$ associated with the noisy precursor of the period doubling bifurcation. Quite analogous behavior is again observed in $NbSe_3$, as shown in Fig. 4. As a single parameter (dc bias) is varied, there appear (at 63.9 mV bias) two bumps in the frequency spectrum, at f' and at $f_0 - f'$, with f_0 = 20 MHz and f' approximately 6 MHz. Between 69.2 mV and 74 mV bias, these two bumps merge into a single bump (period doubling presursor), out of which the true period two peak emerges at 10 MHz (bottom trace in Fig. 4). We tentatively ascribe the behavior of Fig. 4 as representing a virtual Hopf phenomenon.

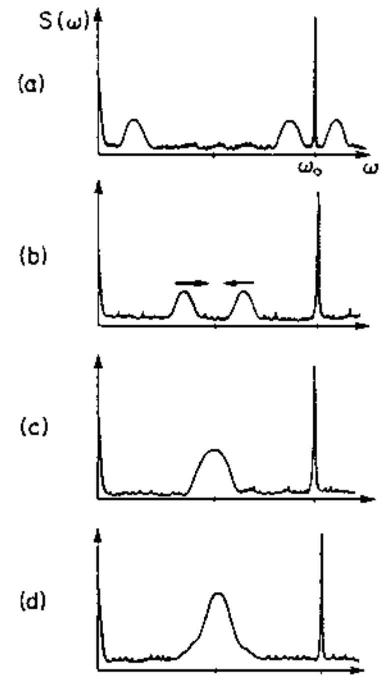

FIGURE 3
Expected spectrum for a virtual Hopf phenomenon. As a parameter is varied, the precursor characteristic of a Hopf bifurcation (a) changes into the precursor of a period-doubling instability (d). From ref. 12

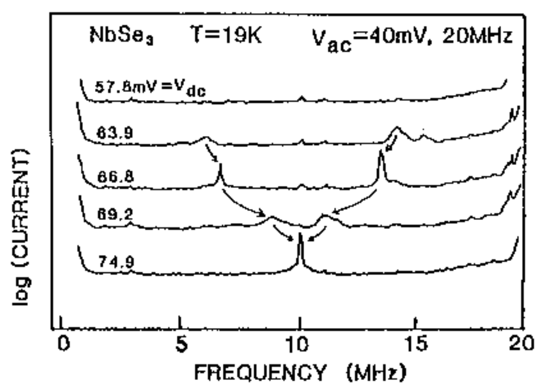

FIGURE 4
Virtual Hopf phenomenon in $NbSe_3$. The parameter being varied is dc bias voltage.

Noisy precursor effects yield quantitative information about the dynamics of the unstable system. For example, the width of the bumps in the virtual Hopf phenomenon reflect a characteristic damping time for perturbations of the periodic orbit. From the width $\Delta f' = 0.8$ MHz in Fig. 4, we find a characteristic relaxation time of approximately 1.25 μsec. This time constant is different from that associated with the CDW pinning mode.

4. CONCLUSION

CDW systems show an interesting variety of chaotic response. The chaos is intimately tied to phase *and* amplitude dynamics of the condensate. A good deal of information about the structure of the CDW system can be obtained by careful analysis of not only the chaotic response, but also noisy precursor effects. It should also be possible to exploit the instability of a CDW system near a period doubling bifurcation to achieve parametric amplification.

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REFERENCES

1. M. Sherwin, R. Hall, and A. Zettl, Phys. Rev. Lett. 53 (1984), 1387.
2. R.P. Hall, M. Sherwin, and A. Zettl, Phys. Rev. Lett. 52 (1984), 2293.
3. R.P. Hall, M. Sherwin, and A. Zettl, Phys. Rev. B29 (1984), 7076.
4. A. Zettl and G. Grüner, Phys. Rev. B29 (1984), 755.
5. R.P. Hall and A. Zettl, Phys. Rev. B30 (1984), 2279; S.E. Brown, G. Mozurkewich, and G. Grüner, Phys. Rev. Lett. 54 (1984), 2272.
6. M.H. Jensen, P. Bak, and T. Bohr, Phys. Rev. A30, (1984), 1960; T. Bohr, P. Bak, and M.H. Jensen, Phys. Rev. A30 (1984), 1970.
7. N.P. Ong, G. Verma, and K. Maki, Phys. Rev. Lett. 52 (1984), 663.
8. R.P. Hall, M.F. Hundley, and A. Zettl, Phys. Rev. Lett. 56 (1986), 2399.
9. R.P. Hall, M.F. Hundley, and A. Zettl (this volume); R.P. Hall and A. Zettl (unpublished).
10. M. Inui, S. Doniach, R.P. Hall, and A. Zettl (unpublished).
11. K. Wiesenfeld, J. Stat. Phys. 38 (1985), 1071.
12. K. Wiesenfeld, Phys. Rev. A32 (1985), 1744.