Homework #5

1. Kittel, Ch4-2. *Surface temperature of the Sun.* The value of the total radiant energy flux density at the earth from the Sun normal to the incident rays is called the solar constant of the Earth. The observed value integrated over all emission wavelengths and refereed to the mean Earth-Sun distance is:

\[
\text{solar constant} = 0.136 \text{ J s}^{-1} \text{ cm}^{-2}.
\] (49)

(a) Show that the total rate of energy generation of the Sun is \(4 \times 10^{26} \text{ Js}^{-1}\). (b) From this result and the Stefan-Bolzmann constant \(\sigma_b = 5.67 \times 10^{-12} \text{ Js}^{-1} \text{ cm}^{-2} \text{ K}^{-4}\), show that the effective temperature of the surface of the Sun treated as a black body is \(T \approx 6000K\). Take the distance of the Earth from the Sun as \(1.5 \times 10^{13} \text{ cm}\) and the radius of the Sun as \(7 \times 10^{10} \text{ cm}\).

2. Kittel, Ch4-5. *Surface temperature of the Earth.* Calculate the temperature of the surface of the Earth on the assumption that as a black body in thermal equilibrium it reradiates as much thermal radiation as it receives from the Sun. Assume also that the surface of the Earth is at a constant temperature over the day-night cycle. Use \(T_e = 5800K; R_e = 7 \times 10^{10} \text{ cm}\); and the Earth-Sun distance of \(1.5 \times 10^{13} \text{ cm}\).

3. Kittel, Ch4-7. *Free energy of photon gas.* (a) Show that the partition function of a photon gas is given by

\[
Z = \prod_n \left[1 - \exp\left(-\frac{\hbar \omega_n}{\tau}\right)\right]^{-1}.
\] (53)

where the product is over the modes \(n\). (b) The Helmholtz free energy is found directly from (53) as

\[
F = \tau \sum_n \log[1 - \exp\left(-\frac{\hbar \omega_n}{\tau}\right)].
\] (54)

Transform of the sum to an integral; integrate by parts to find

\[
F = -\frac{\pi^2 V \tau^4}{45 \hbar^3 c^3}.
\] (55)
4. Kittel, Ch4-11. *Heat capacity of solids in high temperature limit.* Show that in the limit of \( T >> \theta \) the heat capacity of a solid goes towards the limit of \( C_v \rightarrow 3Nk_B \), in conventional units. To obtain higher accuracy when \( T \) is only moderately larger than \( \theta \), the heat capacity can be expanded as a power series in \( 1/T \), of the form

\[
C_v = 3Nk_B \left[ 1 - \sum_n \frac{a_n}{T^n} \right].
\]  (56)

Determine the first nonvanishing term in the sum. Check your result by inserting \( T = \theta \) and comparing with Table 4.2.

5. Kittel, Ch4-14, *Heat capacity of liquid \(^4\)He at low temperature.* The velocity of longitudinal sound waves in liquid \(^4\)He at temperatures below 0.6K is \( 2.383 \times 10^4 \text{ cms}^{-1} \). There are no transverse sound waves in the liquid. The density is \( 0.145 \text{ gcm}^{-3} \). (a) Calculate the Debye temperature. (b) Calculate the heat capacity per gram on the debye theory and compare with the experimental value \( C_v = 0.0204 \times T^3 \), in \( \text{ J g}^{-1} \text{K}^{-1} \). The \( T^3 \) dependence of the experimental value suggests that phonons are the most important excitations in liquid \(^4\)He below 0.6K. Note that the experimental value has been expressed per gram of liquid. The experiments are due to J. Wiebes, C. G. Niels-Hakkenberg, and H. C. Krammers, *Physca* 32, 625 (1957).

6. Kittel, Ch4-15, *Angular distribution of radiant energy flux.* (a) Show that the spectral density of radiant energy flux that arrives in the solid angle \( d\Omega \) is \( \omega u_\omega \cos \theta \cdot d\Omega / 4\pi \), where \( \theta \) is the angle the normal to the unit area makes with the incident ray, and \( u_\omega \) is the energy density per unit frequency range. (b) Show that the sum of this quantity over all incident rays is \( \frac{1}{4} cu_\omega \).