Homework 9

1. (Kittel 7.7) **Photon Condensation.** Consider a science fiction universe in which the number of photons $N$ is constant, at a concentration of $10^{20} \text{ cm}^{-3}$. The number of thermally excited photons we assume is given by the result of Problem 4.1, which is $N_e = 2.404 \frac{V \tau^3}{\pi^2 \hbar^2 c^3}$. Find the critical temperature in K below which $N_e < N$. The excess $N - N_e$ will be in the photon mode of the lowest frequency; the excess might be described as a photon condensate in which there is a large concentration of photons in the lowest mode. In reality there is no such principle that the total number of photons be constant, hence there is no such photon condensate.

2. (Kittel 7.8) **Energy, heat capacity and entropy of degenerate boson gas.** Find expressions as a function of temperature in the region $\tau < \tau_E$ for the energy, heat capacity and entropy of a gas of N noninteracting bosons of spin zero confined to a volume $V$. Put the definite integral in dimensionless form; it need not be evaluated. The calculated heat capacity above and below $\tau_E$ is shown in Figure 7.19. The experimental curve was shown in Figure 7.12. The difference between the two curves is marked: It is ascribed to the effect of interactions between the atoms.

3. (Kittel 7.9) **Boson gas in one dimension.** Calculate the integral for $N_e(\tau)$ for a one dimensional gas of noninteracting bosons, and show that the integral does not converge. This result suggests that a boson ground state condensate does not form in one dimension. Take $\lambda = 1$ for the calculation. (The problem should really be treated by means of a sum over orbitals on a finite line.)

4. (Kittel 7.14) **Two orbital boson system.** Consider a system of N bosons of spin zero, with orbitals at the single particle energies 0 and $\varepsilon$. The chemical potential is $\mu$ and the temperature is $\tau$. Find $\tau$ such that the thermal average population of the lowest orbital is twice the population of the orbital at $\varepsilon$. Assume $N \gg 1$ and make what approximations are reasonable.